Partial Differential Equations

**Problem 1.** For the given equation

\[ xyz_x + xzz_y = yx \]

find a curve s.t. there exists infinitely many solutions passing through this curve.

**Problem 2.** Using Green’s first identity

\[
\int_{\partial D} v \frac{\partial u}{\partial n} dS = \int \int_D \nabla v \cdot \nabla u \, dx \, dy + \int \int_D v \Delta u \, dx \, dy
\]

show that the Dirichlet problem

\[
\begin{align*}
\Delta u + ku &= q(x, y), \quad \text{in } D \\
u &= f(x, y) \quad \text{on } \partial D
\end{align*}
\]

has a unique solution if \( k < 0 \).

**Problem 3.** For equation

\[
\Delta u(x) + u(x) = 0 \quad x \in \mathbb{R}^3
\]

find the spherically symmetric solution that is solution of the form \( u = f(r) \), where \( r = \sqrt{x_1^2 + x_2^2 + x_3^2} \).

**Problem 4.** Let \( \Omega \) be a bounded domain in \( \mathbb{R}^2 \). Suppose that \( u \in C^2(\Omega) \) satisfies

\[
-\Delta u + \nabla u + e^{xy}u > 0 \quad \text{on } \Omega,
\]

and \( u \) assumes a minimum value on \( \Omega \) at some interior point \( x_0 \in \Omega \). Prove that \( u(x_0) \geq 0 \).