METU MATHEMATICS DEPARTMENT
REAL ANALYSIS
SEPTEMBER 2012 - TMS EXAM

1. Prove disprove:

a) If $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable then the improper integral $\displaystyle\int_{-\infty}^{\infty} f(x)\ dm(x)$ is convergent.

b) If $\int_{-\infty}^{\infty} f(x)\ dm(x)$ is convergent then $f \in L^1$.

2. Compute $\lim_{n\to\infty} \displaystyle\sum_{k=0}^{\infty} \left(\frac{n}{2n+k}\right)^k$

(Hint: Use a convergence theorem)

3. Let $E \subset [0,1] \times [0,1]$ have the property that every horizontal section Ey is countable and every vertical section Ex has countable complement $[0,1] \setminus E_x$. Prove that E is not L-measurable.

4. Let (X, σ, μ) be a measure space.

a) Define convergence in measure

b) Let $\phi : \mathbb{C} \to \mathbb{C}$ be uniformly continuous. Let $f_n, f : X \to C$ be measurable and $f_n \to f$ in measure.

Show that $\phi \circ f_n$ converges to $\phi \circ f$ in measure.