Real Analysis Preliminary Exam
Sep 2024

1. (25 pts) Suppose $0 < p_0 < p_1 < \infty$. Find an example of function f on $(0,\infty)$ (with Lebesgue measure) such that $f \in L^p$ iff $p_0 < p < p_1$.

2. (25 pts) Let E be Lebesgue measurable and $m(E) > 0$. Show that for any $\alpha < 1$ there is an open interval I such that $m(E \cap I) > \alpha m(I)$.

3. (10+15pts)

   a) State Lebesgue Dominated Convergence Theorem.

   b)Compute $\lim_{n\to\infty} \int_0^\infty n\sin(x/n)[x(1+x^2)]^{-1}dx$.

4. (25pts) Let $f(x,y) = \begin{cases} x^{-13/10}\cos(\frac{1}{xy}) & \text{if } 0 < y < x < 1 \\ 0 & \text{Otherwise.} \end{cases}$

   Is the following true? Justify your answer.

$$\int_0^1 \int_0^1 f(x,y)dxdy = \int_0^1 \int_0^1 f(x,y)dydx.$$