1. (25 pts) Prove that the set
\[ E = \{ x \in [0, 1] : \forall q \in \mathbb{N}, \exists p \in \mathbb{N} \text{s.t} \ x - \frac{p}{q} \leq \frac{1}{q^2} \} \]
is Lebesgue measurable and its Lebesgue measure is 0.

2. (10+10+5 pts) Let \( m \) be the Lebesgue measure on \([0, 1]\), and \( \mu \) is the counting measure on \([0, 1]\).
   (a) Is \( m \) absolutely continuous with respect to \( \mu \)?
   (b) Does the Radon-Nikodym derivative \( \frac{dm}{d\mu} \) exist?
   (c) Does your result in part (b) contradict to Radon-Nikodym Theorem?

3. (25pts) Let \( f \in L^1(\mathbb{R}) \) and \( g \in L^1(\mathbb{R}) \). Show that \( \int_{\mathbb{R}} |f(x - y)g(y)|dy < \infty \) for a.e \( x \).

4. (25pts) Show that if \( \{f_n\} \in L^+ \), \( f_n \) decreases pointwise to \( f \) and \( \int f_k < \infty \) for some \( k \) then \( \int f = \lim \int f_n \).