M.E.T.U

Department of Mathematics

Preliminary Exam - Sep. 2011

Topology

Duration : 3 hr.

Each question is 25 pt.

1. For a subset A in topological space X, let $\overline{A}$ denote the closure and A° denote the interior of A in X. Prove or disprove the followings:

   (a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

   (b) $(A \cup B)^\circ = A^\circ \cup B^\circ$.

   (c) $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

   (d) Every quotient space of a Hausdorff space is Hausdorff.

   (e) Every quotient map $p : X \to Y$ is open.

   (f) An infinite set X with the finite complement topology is metrizable.

   (g) $(-\infty, 0)$ is homeomorphic to $(0, 1)$.

   (h) $(-\infty, 0)$ is homeomorphic to $[0, 1)$.

2. (a) Let X be a topological space and let A be a subset of X. Give the definition of the connectedness and the path-connectedness of A.

   (b) Prove that a path-connected subset of a topological space is connected.

   (c) Show that the converse of (b) is not true.

   (d) Let X be a topological space, C be a connected subset and E be an arbitrary subset of X. Suppose that $C \cap E \neq \emptyset$ and $C \cap (X - E) \neq \emptyset$. Show that $C \cap \partial E \neq \emptyset$, where ∂E denotes the boundary of E.

   (e) Let D denote the subset $\{(x_1, x_2, \ldots, x_n, 0) : x_1^2 + x_2^2 + \cdots + x_n^2 \neq 1\}$ in $\mathbb{R}^{n+1}$. Is D connected? Prove your answer.

3. Prove the following:

   (a) If X is a compact space and $f : X \to Y$ is a continuous surjective map, then Y is compact.

   (b) Let X be a compact space and Y be a Hausdorff space. Suppose that $f : X \to Y$ is a continuous bijection. Prove that f is a homeomorphism.

   (c) Prove that the compactness assumption in (b) is necessary.

4. Let X, Y be Hausdorff topological spaces. Prove the followings:

   (a) Prove that $X \times Y$ is compact if and only if X and Y are compact.

   (b) Prove that $X \times Y$ is path–connected if and only if X and Y are path–connected.