Topology **September 2013** **Middle East Technical University**

Preliminary Exam
DURATION: 3 hours

1- Prove or disprove. (X, Y are topological spaces, A, B are subsets of a topological space X, $\overline{A}$ denotes the closure of the set A, A' denotes the set of limit points of the set A, A° denotes the interior of the set A, ∂A denotes the boundary of the set A.)

(a) $(A \cup B)^\circ = A^\circ \cup B^\circ$.

(b) $f^{-1}(C') = (f^{-1}(C))'$ for any continuous function $f : X \to Y$ and for all $C \subset Y$.

(c) If $A^\circ \neq \emptyset$, then $\overline{A^\circ} = \overline{A}$.

(d) If A and B are connected and $A \cap B \neq \emptyset$, then $A \cap B$ is connected.

(e) If X is connected and if A is a proper subset of X (that is $A \neq \emptyset$ and $A \neq X$), then $\partial A \neq \emptyset$.

2- Let

$$\mathcal{T} = \{(-\infty, a) \mid a \in \mathbb{R}\}.$$

(a) Show that $\mathcal{T}$ is a topology on $\mathbb{R}$.

(b) Compare this topology with the standard topology on $\mathbb{R}$.

(c) Let $A = (-1, 1)$ and $B = (-\infty, 1]$. Find the interiors A°, B° and the closures $\overline{A}$, $\overline{B}$ of the sets A, B in this topology.

3- Let X, Y be topological spaces where Y is Hausdorff. Let $A \subset X$ be dense in X, i.e. $\overline{A} = X$. Let $f, g : X \to Y$ be continuous functions such that $f(a) = g(a)$ for all $a \in A$. Show that $f = g$.

4- Let X, Y be topological spaces, $a \in X$ and $C \subset Y$ be compact in Y. Suppose there is an open set N in $X \times Y$ such that $\{a\} \times C \subset N$. Show that there is an open set $U \subset X$ and an open set $V \subset Y$ such that $a \in U$, $C \subset V$ and $U \times V \subset N$.