Topology, February 2012
TMS EXAM
February 15, 2012

1. Let $\mathcal{T} = \{U \cup -U \mid U \subset \mathbb{R}$ is an open set in standard topology of $\mathbb{R}\}$. Show that $\mathcal{T}$ is a topology on $\mathbb{R}$ and find the interiors and the closures of the following intervals of $\mathbb{R}$ in this topology: $A = (-2, 3)$, $B = (-\infty, 3)$, $C = (1, 3)$.

2. Let X be a topological space. Prove the given statement or give a counter example to show that it is not always true.

a) If $A \subset X$ is connected then $\overline{A}$ is also connected.

b) If $A \subset X$ is path connected then $\overline{A}$ is also path connected.

3. Consider the space $X = \{a, b, c\}$ with the topology $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

a) Show that $Y = \{1, 2\}$ with topology $\{\emptyset, Y, \{1\}, \{2\}\}$ is not a quotient space of X.

b) Show that $Y = \{1, 2\}$ with topology $\{\emptyset, Y, \{1\}\}$ is a quotient space of X.

c) Consider the function $f : X \to Y$, $f(a) = 1$, $f(b) = 1$ and $f(c) = 2$, where Y has the indiscrete topology $\{\emptyset, Y\}$. Is f a quotient function? Is Y is a quotient space of X?

4. Let X be a countable set (you may take, for example, $X = \mathbb{Z}_+$) equipped with the discrete topology. Find a homeomorphism $f : X^+ \to A$, where A is the subspace of $\mathbb{R}$ given by

$$A = \{\frac{1}{n} \mid n = 1, 2, \cdots\} \cup \{0\}$$

and X^+ is the one point compactification of X.