Topology   February 2013   Middle East Technical University

Preliminary Exam
DURATION: 3 hours

1- Prove or disprove. (X, Y are topological spaces, A, B are subsets of a topological space X, $\overline{A}$ denotes the closure of the set A, A' denotes the set of limit points of the set A.)

(a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(b) $f(A') = f(A)'$ for any continuous function $f : X \to Y$.

(c) If X and Y are Hausdorff, then $X \times Y$ is Hausdorff in the product topology.

(d) If X is a metric space then X is Hausdorff.

(e) A infinite set X with the finite complement topology (that means a set $U \subset X$ is open if $X - U$ is finite) is metrizable.

2- For a point $x = (x_1, x_2) \in \mathbb{R}^2$ and a positive number $r > 0$, define

$$B(x,r) = \{y = (y_1, y_2) \in \mathbb{R}^2 \mid |x_1 - y_1| < r\}.$$

Let $\mathcal{B} = \{B(x,r) \mid x \in \mathbb{R}^2, r > 0\}$.

(a) Show that $\mathcal{B}$ is a basis for a topology on $\mathbb{R}^2$

(b) Compare this topology with the standard topology on $\mathbb{R}^2$.

(c) Let $D = \{x = (x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < 1\}$. Find the interior D° and the closure $\overline{D}$ of the set D in this topology.

3- Let $\pi : X \times Y \to X$ be the projection map onto the first coordinate. Show that if Y is compact then π is a closed map.

4- Let X, Y be metric spaces. Using the definition of continuity ONLY, show that a function $f : X \to Y$ is continuous if and only if $f(x_n) \to f(x)$ whenever $x_n \to x$.