Name:

Topology Feb 2018

1. Consider the set $\mathbb{R}$ of real numbers with the topology $\tau = \{(a, \infty) : a \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}$.

   (a) Determine whether $(-\infty, 0)$ and $(0, 3)$ are compact.

   (b) Find the closure and the interior of the set $A = [-1, 1]$.

   (c) Let $x_n = 2$ for all $n = 1, 2, 3, \ldots$. Find all limits of the sequence (x_n).

2. Prove or disprove the followings:
   (a) Every quotient of a Hausdorff space is Hausdorff.
   (b) Every quotient of a path-connected space is path-connected.

3. Let X and Y be two topological spaces, $x_0 \in X$ and let W be a (open) neighborhood of $x_0 \times Y$ in $X \times Y$.
   (a) Prove that if Y is compact, then there is a neighborhood N of x_0 in X such that $N \times Y \subset W$.
   (b) Show that the conclusion of (a) may not hold if Y is not compact.

4. Let X and Y be two topological spaces and $f, g : X \to Y$ be continuous functions. Suppose that Y is Hausdorff and that there exists a dense subset D of X such that $f(d) = g(d)$ for all $d \in D$. Prove that $f(x) = g(x)$ for all $x \in X$