

**METU Complex Analysis Preliminary Exam**  
**Feb 2026**

$\mathbb{D} = \{z : |z| < 1\}$  denotes the unit disc in all problems here. Show your work.

1. (25 pts) Let  $f$  be an entire function such that  $f(\frac{1}{n}) = \sin(\frac{1}{n})$  for all integers  $n \geq 1$ . What can you conclude about  $f$ ? Explain.

2. (25 pts) Find the number of zeros of  $f(z) = z - 3 + e^{-z}$  on the right half plane.

3. (25 pts) Construct a conformal mapping from

$$S = \{z \in \mathbb{C} : |z| > 1\} \setminus (-\infty, -1]$$

to the unit disc  $\mathbb{D}$ .

4. (25 pts) Compute  $\int_{|z|=1} \frac{\sin z}{z^3 \cos z} dz$ .