

TMS. Differential Equations (ODE)

1. Solve the equation,

$$\frac{dy}{dx} = \cos(x - y - 1). \quad (1)$$

2. Consider the equation,

$$\frac{dy}{dx} = y^3 + f(x), \quad (2)$$

where $f(x)$ is an ω -periodic continuous function.

Prove that

$$\int_0^\omega (y_1^2(x) + y_1(x)y_2(x) + y_2^2(x))dx = 0,$$

if y_1, y_2 are two different ω -periodic solutions of the equation (2).

3. Analyze stability of the equilibria of the equation,

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \sin(x) = 0, k > 0. \quad (3)$$

4. Use the second Lyapunov method (the method of Lyapunov functions) to prove stability of the trivial solution, $x = 0, y = 0$, of the system,

$$\frac{dx}{dt} = y, \frac{dy}{dt} = -f(x), \quad (4)$$

if $f(0) = 0, xf(x) > 0, x \neq 0$.

Hint: Use the function $\frac{y^2}{2} + \int_0^x f(t)dt$.