

TMS. Differential Equations (PDE)

1. (a) Find the general solution of the equation

$$\frac{1}{4}u_{ttt} + u_{tt} = 0 \tag{1}$$

(b) Determine the solution of (1) such that $u(x, 0) = \sin x, u'_t(x, 0) = \cos x, u''_{tt}(x, 0) = \arctan x$.

2. Use the method of characteristics to solve the following problems:

(a) $u_t + xu_x = 0, u(x, 0) = \psi(x)$.

(b) $u_t + cu_x = u^2, u(x, 0) = \cos(x)$.

3. Apply the method of separation of variables to solve the problem

$$\begin{aligned} u_{tt} &= u_{xx} + 2u_x, \\ u(0, t) &= 2e^t - 3e^{-t}, t \geq 0. \end{aligned}$$

4. Consider the equation $u_{xx} - 2xyu_{yy} = 0$.

(a) Find and picture domains in R^2 , where the above equation is parabolic, elliptic, hyperbolic.

(b) Reduce the above equation to canonical forms in all domains determined in (a).

(c) Use the result in (b) to reduce the equation $u_{xx} - 2xyu_{yy} + 5xyu + e^x y = 0$ to the canonical form, if $x > 0, y < 0$.