Department of Mathematics METU
Topology TMS Exam
February 13, 2026

Problem 1. (a) Decide which pairs of spaces are homeomorphic or or not, prove your claims:
(i) $X = \mathbb{R}^n - \{0\}$ and $Y = \mathbb{R}$, $n \geq 2$,
(ii) $U = \mathbb{Z}$ and $V = \mathbb{Z} \times \mathbb{Z}$,
All sets are given the standard topology (as subspaces of $\mathbb{R}^n$).

(b) Let X be a topological space and let $\mathcal{B} = \{B_\alpha\}_{\alpha \in I}$ be a collection of nonempty open subsets that is a base for the topology on X. For each $\alpha \in I$, let $x_\alpha \in B_\alpha$. Prove that $A = \{x_\alpha\}_{\alpha \in I}$ is a dense subset of X.

Problem 2.
(a) State the following notions for a topological spaces X and Y:
(i) X is compact
(ii) X is Hausdorff
(iii) Y is a quotient of X
(b) Suppose that X is a compact Hausdorff topological space and $A \subset X$ is closed. Prove that X/A is compact and Hausdorff.

Problem 3.
(a) Let

$$A = \{(x, y) \in \mathbb{R}^2 \mid x > 0, y \geq 0 \text{ and } \frac{y}{x} \text{ is rational}\}$$

and equip A with the subspace topology induced by the usual topology on $\mathbb{R}^2$. Prove or disprove that A is connected.
(b) Determine whether $\mathbb{Z}$ with the topology $\mathcal{T} = \{\emptyset, \mathbb{Z}\}$ is path-connected.

Problem 4. Let X and Y be topological spaces and let $f : X \to Y$ be a function. Suppose that $X = A \cup B$ where A and B are closed subsets, and that the restrictions $f|_A : A \to Y$ and $f|_B : B \to Y$ are continuous (where A and B have the subspace topology). Prove that f is continuous.